Random phase approximation implies nearly homogeneous wave turbulence.

Peter A.E.M. Janssen
E.C.M.W.F. Shinfield Park, Reading, U.K.

Abstract

In this presentation I would like to discuss why, assuming that the sea state is close to Gaussian statistics as follows from the Random Phase Approximation (RPA), wave turbulence is almost homogeneous.

My starting point is the random version of the Nonlinear Schrödinger (NLS) equation as derived by Alber and Saffman (1978). This is an evolution equation for the two-point correlation function

$$\rho(x, r, t) = \langle A(x_1, t) A^*(x2, t) \rangle \tag{1}$$

where the average coordinate x is defined by $x = (x_1 + x_2)/2$ while the separaton coordinate is $r = x_2 - x_1$. For an inhomogeneous wave field the correlation function depends on both r and x while for homogeneous wave field ρ only depends on r.

Alber and Saffman have shown that a homogeneous wave field is unstable to inhomogeneous perturbations provided the Benjamin-Feir index is larger than 1 ($BFI > 1$).

One would therefore expect that this instability (called the random version of the Benjamin-Feir instability would generate an inhomogeneous ensemble of ocean waves.

Using numerical experiments it is shown that for a wave system satisfying RPA this is not the case. Initially there is a small amount of inhomogeneity generated that in the course of time decays to zero. The reason for this is that there is a quasi-linear interaction between

the homegeneous wave field and the unstable imhomogeneous perturbations which results in a broadening of the homegeneous wave spectrum is such a way that the BFI reduces and becomes slightly smaller than 1 hence quenching the random version of the Benjamin-Feir instability. Therefore, even narrow-band nonlinear ocean waves may be well-described by the homogeneous formalism suggested by Hasselmann in 1960 and extended by Janssen (2003) to include quasi-resonant four-wave interactions.