Ordinary Differential Equations - 10413181

Homework No. 10

1. Find two linearly independent solutions to the equation

$$t^2 y'' - 2y = 0$$

   using the *ansatz* $y = t^r$. Then, using the two solutions you have found, find the

   general solution to the inhomogeneous equation

$$t^2 y'' - 2y = t^2$$

   using your favourite method.

2. Determine the radius of convergence of the following power series:

   (a) $\sum_{n=0}^{\infty} (x-2)^n$

   (b) $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$

   (c) $\sum_{n=0}^{\infty} \frac{(2x+1)^n}{n^2}$

3. For the following ODE, solve using expansion in power series about $x_0 = 0$

$$y'' - xy' - y = 0.$$

   Find at least terms to 4th order (also the general term, if possible).

   Substitute a truncated version of your solution

$$\psi = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

   into the equation. What is the order of the lowest-order term remaining?