Ordinary Differential Equations - 10413181

Homework No. 11

1. Use power series expansion to find the first four terms of

$$y'' + \cos(x)y = 0.$$

2. **Since no one knew this in class...** Show for $|x| < 1$ that

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}.$$

Hint: the series on the left is defined as the limit of partial sums $\lim_{N\to\infty} \sum_{n=0}^{N} x^n$. *Can you use the denominator of the right-hand side to simplify this and prove the identity?*

3. Transform the following equations into first order systems:

   (a) $u'' + 2u' + 2u = 0$

   (b) $t^2u'' + tu' + (t^2 - 1)u = 0$

   (c) $u^{(4)} - u = 0$

4. Solve the following system of equations

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 7 \\ -x_1 + x_2 - 2x_3 &= -5 \\ 2x_1 - x_2 - x_3 &= 4 \end{aligned}$$

5. Find eigenvectors and eigenvalues for the following matrices

$$A = \begin{pmatrix} 3 & -1 \\ 4 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$$

Hint: characteristic polynomial...