Ordinary Differential Equations - 10413181

Homework No. 12

1. Consider the vectors

$$\mathbf{x}^1(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}, \mathbf{x}^2(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}.$$

   (a) Compute the Wronskian of $\mathbf{x}^1$ and $\mathbf{x}^2$.

   (b) In what intervals are these two vectors linearly independent?

2. Consider the vectors

$$\mathbf{x}^1(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}, \mathbf{x}^2(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$$

   and answer the questions posed in 1.

3. Given the system

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \mathbf{x}$$

   are

$$\mathbf{x}^1(t) = e^{2t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{x}^2(t) = e^{-t} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{x}^3(t) = e^{-t} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

   a fundamental set of solutions to the system? Write down the general solution.

4. Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 4 \\ -2 \end{pmatrix}.$$

   Remember from class: if the eigenvalues λ_i are real and different, two fundamental solutions to this system are $\mathbf{x}^1 = \xi^1 e^{\lambda_1 t}$ and $\mathbf{x}^2 = \xi^2 e^{\lambda_2 t}$, for eigenvectors ξ^1 and ξ^2 to those eigenvalues.