Ordinary Differential Equations - 10413181

Homework No. 4

1. In class we learned about a special type of equation called a **Bernoulli equation**, which has the general form

$$y' + a(x)y = b(x)y^n, \quad n \in \mathbb{R}.$$

You know how to solve this equation for $n = 0$ and $n = 1$ by general methods. It was shown that a transform $y^{1-n} = v$ can be used to reduce such an equation to a more manageable form. Employ this for the following problems:

(a) $y' - y = y^2$

(b) $y' = \frac{2}{x}y + \frac{x}{y^2}$

(c) $t^2y' + 2ty - y^3 = 0$

2. Another special type of equation is known as a **Riccati equation,** with general form

$$y' = q_1(t) + q_2(t)y + q_3(t)y^2.$$

(Note, if $q_1 = 0$ this is just a Bernoulli equation; see if you can spot the commonality in the methods for solving them. Hint: you can always find a certain particular solution to the Bernoulli equation.) If a particular solution $y_1(t)$ of this equation is known (for example, through educated guesswork), a more general solution can be found via the substitution

$$y(t) = y_1(t) + \frac{1}{v(t)}.$$

Try this on the following equations:

(a) $y' = 1 + t^2 - 2ty + y^2$, with particular solution $y_1(t) = t$

(b) $y' = -\frac{1}{t^2} - \frac{y}{t} + y^2$ with particular solution $y_1(t) = 1/t$

3. We discussed the matter of existence and uniqueness of solutions in class. Given the initial value problem

$$y' = y^{1/3}\sin(2t), \quad y(0) = 0$$

(a) Find a trivial solution.

(b) Setting aside the fact that $y(0) = 0$ (you may assume $y(0) = \varepsilon$ and consider the limit, or simply temporarily ignore this) find two other solutions.

(c) Check the assumptions of the existence & uniqueness theorem to explain this multiplicity of solutions.