Ordinary Differential Equations - 10413181

Homework No. 6

1. For the following equations, determine whether the given functions are solutions, and whether they are a fundamental set of solutions (*reminder: Wronskian*).

   (a) $y'' + 4y = 0, \quad y_1(t) = \cos(2t), y_2(t) = \sin(2t)$

   (b) $x^2y'' - x(x+2)y' + (x+2)y = 0$, $(x > 0), \quad y_1(x) = x$, $y_2(x) = xe^x$

2. For the equation

$$y'' - y' - 2y = 0$$

   show that $y_1 = e^{-t}$, $y_2 = e^{2t}$ are a fundamental set of solutions.

   (a) For $y_3 = -2e^{2t}$, $y_4 = y_1 + 2y_2$, $y_5 = 2y_1 - 2y_3$, are y_3, y_4, and y_5 also solutions of the ODE?

   (b) Which of the following pairs forms a fundamental set of solutions?

   (y_1, y_3), (y_2, y_3), (y_1, y_4), (y_4, y_5)

3. Determine the Wronskian of two solutions of the following equation (*Hint: you do not need to solve the equation.*)

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0$$

4. Find the general solution to the following equation:

$$y'' - 2y' + 2y = 0$$