Ordinary Differential Equations - 10413181

Homework No. 7

1. For the following expressions (with $t \in \mathbb{R}$), identify the real part, the imaginary part, the complex conjugate, and use Euler's formula to write each as $a + ib$

   (a) $\exp((1+2i)t))$

   (b) $\exp((2+4i)t))$

   (c) $e^{2+i\pi/2}$

2. Find solutions to the following initial value problems, write these using Euler's formula, and describe their behavior for large time t

   (a) $y'' + 4y = 0, \quad y(0) = 0, y'(0) = 1$

   (b) $y'' + y' + 5y = 0, \quad y(0) = 1, y'(0) = 0$

   (c) $y'' - 6y' + 13y = 0, \quad y(\pi/2) = 0, y'(\pi/2) = 2$

3. (Optional) Pick one of the problems above and determine the envelope and (pseudo-) frequency of the oscillation, if appropriate.

4. In class we saw that certain kinds of second-order ODEs with non-constant coefficients can be reduced to the constant coefficient case with a suitable transformation. For the Cauchy-Euler type equation

$$t^2 y'' + 4ty' + 2y = 0$$

   use the substitution $x = \ln(t)$ to calculate dy/dt, d^2y/dt^2 and solve the equation.