Ordinary Differential Equations - 10413181

Homework No. 9

1. Given the following inhomogeneous second order equation

$$y'' - y' - 2y = 2e^{-t}$$

   (a) Solve by comparing coefficients. (*Make an ansatz for a particular solution* $\psi = \ldots$. *If something goes wrong initially, try solving the homogeneous problem first to identify the origin of your problems. Then try* $\psi = t \cdot \ldots$)

   (b) Use variation of parameters to solve the same problem.

   (c) Finally, given one solution y_1 of the homogeneous problem, use reduction of order with the ansatz $y = y_1(t)v(t)$ to derive a first order equation for $v'(t) = u(t)$.

   (d) Show that finding a particular solution to the related problem

$$y'' - y' - 2y = 2e^{-t} + 1$$

   is equivalent to adding a particular solution of $y'' - y' - 2y = 1$ to a particular solution of $y'' - y' - 2y = 2e^{-t}$. *You may use any of the above methods to determine the particular solutions.*

2. In class, we considered the case of damped harmonic oscillators

$$my'' + \gamma y' + ky = 0$$

   where m is mass, γ a damping coefficient, and k a spring constant, and m, $k > 0$, $\gamma \geq 0$. Assume a motor is attached to the damped spring which supplies an oscillatory force $F_0 \cos(\omega t)$, so that

$$my'' + \gamma y' + ky = F_0 \cos(\omega t).$$

   (a) We would like to find a particular solution to this general equation. Think first about which method is appropriate, and then use that method to obtain such a solution.

   (b) Look up the form of the homogeneous solutions from class (or derive them yourself). What is the long–time behavior ($t \to \infty$) of the general solution?

   (c) (Optional) Analyze the above problem for $\gamma = 0$. Does any particular case of ω in the forcing play a special role? (*We will discuss this in class.*)