Homework 2 Solutions

1. (a) $y' + \tan(x)y = x\sin(2x)$ invites one to apply the integrating factor method. $\mu = e^{\int \tan(x)dx} = e^{-\ln(\cos(x))}$. Hence

$$\frac{y}{\cos(x)} = \int \frac{x\sin(2x)}{\cos(x)}dx = \int \frac{2x\sin(x)\cos(x)}{\cos(x)}dx$$
$$= 2\int x\sin(x)dx = 2(\sin(x) - x\cos(x))$$

Hence: $y = 2\sin(x)\cos(x) - 2x\cos^2(x) + C\cos(x)$

(b) $y' - 3x^2y = -x^2, \quad y(0) = 1$ again should be treated by the integrating factor method, with $\mu = e^{-x^3}$. Integrating both sides gives

$$y = 1/3 + Ce^{x^3}$$

and the initial value $y(0) = 1 \Rightarrow C = 2/3$.

2. Using either separation or the integrating factor method leads one to the form of solution

$$y = Ce^{-\int \frac{\ln^2(x)}{\sin^2(x)}dx}.$$

The initial condition is $y(5) = 0$, which can only be satisfied if $C = 0$. Hence the integral need not be treated, and the solution $y = 0$ is the unique solution (at least on an interval $[5, 2\pi]$).

3. (a) Separation gives

$$\frac{y'}{y^2} = -\sin(x)$$

hence

$$y = \frac{-1}{\cos(x) + C}$$

(b) Separation again yields

$$\int \frac{1}{1+y^2}dy = 2\int 1 + xdx$$

or $y = \tan(2x + x^2 + C)$, whereupon the initial condition implies $C = 0$.

(a) Making the substitution reduces the ODE to $xv' = 1 + v$ which is separable. Hence

$$\int \frac{dv}{1+v} = \int \frac{dx}{x}$$

so that $\ln|1+v| = \ln|x| + C \Rightarrow |1+v| = |x|e^C \Leftrightarrow v = cx - 1$ for $c \neq 0$. Transforming back to the original variables gives $y = cx^2 - x$ again for $c \neq 0$.

(b) Substitution again reduces the ODE to a separable equation $xv' = 1 + v^2$, so that

$$\int \frac{dv}{1+v^2} = \int \frac{1}{x} dx$$

Hence $\arctan(v) = \ln|x| + C \Rightarrow y = x\tan(\ln|x| + C)$