Homework 2 Solutions

1. Identifying this equation as $M + Ny' = 0$ with

$$M = 2t\sin(y) + y^3 e^t$$
$$N = t^2\cos(y) + 3y^2 e^t$$

we find that

$$M_y = 2t\cos(y) + 3y^2 e^t = N_t$$

Hence $\exists \phi : \nabla\phi = (M, N)$, and

$$\phi = \int M dt + k(y) = \int 2t\sin(y) + y^3 e^t dt + k(y) = t^2\sin(y) + y^3 e^t + k$$
$$\phi = \int N dy + h(t) = \int t^2\cos(y) + 3y^2 e^t dy + h(t) = t^2\sin(y) + y^3 e^t + h$$

Hence the solution curves are given by

$$t^2\sin(y) + y3e^t = C$$

Inspecting

$$\frac{d}{dt}(t^2\sin(y) + y3e^t) = 0$$

recovers the ODE.

2. $y' + y = 5\sin(2t)$ is a candidate for the integrating factor method with $\mu = e^t$. Thus:

$$ye^t = 5\int e^t\sin(2t)dt$$

By using partial integration twice, we find

$$\int e^t\sin(2t)dt = e^t\sin(2t) - \int e^t 2\cos(2t)dt$$
$$= e^t\sin(2t) - \left(e^t 2\cos(2t) + 4\int e^t\sin(2t)dt\right)$$

Hence $5\int e^t\sin(2t)dt = e^t\sin(2t) - e^t 2\cos(2t)$, resulting in

$$y = \sin(2t) - 2\cos(2t) + Ce^{-t}.$$

3. This equation is separable, such that

$$\int \frac{dy}{\cos^2(2y)} = \int \cos^2(x)dx$$

These integrals lead to

$$\tan(2y) = x + \sin(2x)/2 + C$$

for $\cos(2y) \neq 0$. Another solution is $y = \pm(2n+1)\pi/4$ for any integer n.

4. Writing the equation in normal form $y' + y/2 = 3t^2/2$ allows an immediate identification of the integrating factor $\mu = e^{t/2}$. Resolving the integral $\int e^{t/2} t^2 dt$ by partial integration (twice, differentiating the t^2 term) yields

$$y = ce^{-t/2} + 3t^2 - 12t + 24$$

5. This equation is of the type $M + Ny' = 0$ with

$$\begin{aligned} M &= 2x + 3 \\ N &= 2y - 2 \end{aligned}$$

Inspection yields

$$M_y = 2 = N_x$$

hence there is a potential ϕ, such that

$$\begin{aligned} \phi &= \int 2x + 3dx + c(y) = x^2 + 3x + C \\ \phi &= \int 2y - 2dy + d(x) = y^2 - 2y + C \end{aligned}$$

and the solution curves are

$$x^2 + 3x - 2y + y^2 = C.$$

6. The integrating factor is $\mu = e^{-4t}$, which yields (integrating over 1 on the RHS)

$$y = (t + C)e^{4t}$$

the IVP $y(0) = 2$ implies $C = 2$.