PDEs 10422884 – Homework 1

This homework must be handed in prior to the tutorial on ***April 4th, 2017****. Questions marked with $*$ will be graded, and will go towards your grade on the homework. Unmarked questions will be checked for completion (or a reasonable attempt).*

*1. Which of the following operators are linear?

(a) $L[u] = u_x + xu_y$
(b) $L[u] = u_x + uu_y$
(c) $L[u] = u_x + u_y^2$
(d) $L[u] = u_x + u_y + 1$
(e) $L[u] = \sqrt{1+x^2}(\cos(y))u_x + u_{yxy} - (\arctan(x/y))u$

*2. For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous, or linear homogeneous; provide reasons.

(a) $u_t - u_{xx} + 1 = 0$
(b) $u_t - u_{xx} + xu = 0$
(c) $u_t - u_{xxt} + uu_x = 0$
(d) $u_{tt} - u_{xx} + x^2 = 0$
(e) $iu_t - u_{xx} + u/x = 0$
(f) $u_x(1+u_x^2)^{-1/2} + u_y(1+u_y^2)^{-1/2} = 0$
(g) $u_x + e^y u_y = 0$
(h) $u_t + u_{xxxx} + \sqrt{1+u} = 0$

*3. Verify that $u(x,y) = f(x)g(y)$ is a solution of the PDE $uu_{xy} = u_x u_y$ for all pairs of (differentiable) functions f and g of one variable.

4. Verify by direct substitution that

$$u_n(x,y) = \sin(nx)\sinh(ny)$$

is a solution of $u_{xx} + u_{yy} = 0$ for every $n > 0$.

*5. Solve the equation $3u_y + u_{xy} = 0$. (*Hint:* Let $v = u_y$.)

6. Show that the solutions of the differential equation $u''' - 3u'' + 4u = 0$ form a vector space. Find a basis of it.