PDEs 10422884 – Homework 4

This homework must be handed in prior to the tutorial on ***May 11th, 2017****.*

*1. Find the regions in the xy plane where the equation

$$yu_{xx} - 2u_{xy} + xu_{yy} = 0$$

is elliptic, hyperbolic, or parabolic.

2. Reduce the elliptic equation

$$u_{xx} + 3u_{yy} - 2u_x + 24u_y + 5u = 0$$

to the form $v_{xx} + v_{yy} + cv = 0$ by a change of dependent variable $u = ve^{\alpha x+\beta y}$ and then a change of scale $y' = \gamma y$.

*3. Consider the equation

$$u_{xx} + yu_{yy} = 0.$$

Find the canonical form of the equation for the domain where it is hyperbolic.

4. Consider the equation

$$xu_{xx} - yu_{yy} + \frac{1}{2}(u_x - u_y) = 0.$$

Find the domain where the equation is elliptic, and the domain where it is hyperbolic. For each of the two domains, find the corresponding canonical transformation.

**5. (Optional) Reduce the following second-order equation to a system of first-order equations.

$$u_{tt} - 4u_{xt} - 5u_{xx} = 0$$

Use the method of characteristics to find the general solution.