PDEs 10422884 – Homework 6

This homework must be handed in prior to the tutorial on ***June 8th, 2017***.

*1. Solve
$$u_{tt} - c^2 u_{xx} = e^{ax},\ u(x,0) = 0,\ u_t(x,0) = 0.$$

*2. Solve
$$u_{tt} = 4u_{xx}$$
on $0 < x < \infty$, $u(0,t) = 0$, $u(x,0) = 1$, $u_t(x,0) = 0$ using the reflection method. Does your solution have a singularity? Try to explain why, in terms of compatibility of the initial and boundary conditions (*optional*).

*3. Find a solution to the problem
$$u_{tt} - c^2 u_{xx} = 0,\ x > 0,\ t > 0$$
with $u(0,t) = 0$, $t > 0$, and $u(x,0) = xe^{-x}$, $u_t(x,0) = 0$, $x > 0$.

*4. Find a particular solution u_p to the inhomogeneous equation
$$u_{tt} - u_{xx} = t^7,\ -\infty < x < \infty,\ t > 0.$$
Using this particular solution, solve the Cauchy problem with initial data
$$u(x,0) = 2x + \sin(x),\ u_t(x,0) = 0, -\infty < x < \infty$$
by substituting $w = u - u_p$ and using D'Alembert's formula.

**5. (Optional) Solve
$$u_{tt} = c^2 u_{xx},\ 0 < x < \infty,\ 0 \leq t < \infty,\ u(x,0) = 0, u_t(x,0) = V$$
subject to $u_t(0,t) + au_x(0,t) = 0$, where V, a and c are positive constants, and $a > c$.