# PDEs 10422884 – Homework 7

This homework must be handed in prior to the tutorial on June 15th, 2017.

1. Consider waves in a resistant medium satisfying the problem

$$\begin{aligned} &u_{tt} = c^2 u_{xx} - r u_t,\ 0 < x < l \\ &u = 0 \text{ at } x = 0,\ x = l \\ &u(x,0) = \phi(x) \\ &u_t(x,0) = \psi(x) \end{aligned}$$

   for $0 < r < 2\pi c/l$ a constant. Use separation of variables to write down the series expansion of the solution. *Optional:* Repeat this for the case $2\pi c/l < r < 4\pi c/l$.

*2. Use separation of variables to solve the Schrödinger equation $u_t = i u_{xx}$ on the interval $(0, l)$ with Dirichlet conditions on both ends.

3. Solve the diffusion problem $u_t = k u_{xx}$ in $0 < x < l$ with mixed boundary conditions $u(0,t) = u_x(l,t) = 0$.

4. (a) Consider the SLP

$$(x^2 v')' + \lambda v = 0,\ 1 < x < b,\ v(1) = v(b) = 0,\ (b > 1).$$

   Find the eigenvalues and eigenfunctions of the problem. *Hint:* show that

$$v(x) = x^{-1/2} \sin(\alpha \ln x)$$

   is a solution of the ODE and satisfies the boundary condition $v(1) = 0$. Alternatively: recall how to solve Euler ODE's.

   (b) Write down a solution to the following problem using separation of variables

$$\begin{aligned} &u_t = (x^2 u_x)_x \text{ for } 1 < x < b,\ t > 0 \\ &u(1,t) = u(b,t) = 0,\ t \geq 0 \\ &u(x,0) = f(x),\ 1 \leq x \leq b \end{aligned}$$