PDEs Homework 1:

$\boxed{1}$

a) linear

b) not linear operator

c) —— " —— " ——

d) —— " —— " —— $(L[u+v] \neq L[u]+L[v],\ L[0] \neq 0\,!)$

e) linear

$\boxed{2}$

a) 2^{nd} order, inhom., linear

b) 2^{nd} order, hom., linear

c) 3^{rd} order, nonlinear

d) 2^{nd} order, inhom., linear

e) 2^{nd} order, hom., linear

f) 1^{st} order, nonlinear

g) 1^{st} order, hom., linear

h) 4^{th} order, nonlinear

$\boxed{3}$

$$u(x,y) = f(x)g(y) \Rightarrow \left.\begin{aligned} u_x &= f'g \\ u_y &= fg' \\ u_{xy} &= f'g' \end{aligned}\right\} u u_{xy} = fgf'g' = u_x u_y$$

$\boxed{4}$

$$u_n(x,y) = \sin(nx)\sinh(ny)$$

$$u_x = n\cos(nx)\sinh(ny)$$

$$u_{xx} = -n^2\sin(nx)\sinh(ny)$$

$$u_y = \sin(nx)\cdot n\cosh(ny)$$

$$u_{yy} = \sin(nx)\cdot n^2\sinh(ny)$$

$$\Sigma = 0 \quad \checkmark$$

5 $3u_y + u_{xy} = 0 \overset{v=u_y}{\iff} 3v + v_x = 0$

$v' + 3v = 0$ can be solved e.g. by integr. factor

$v = Ce^{-3x}$ $\Rightarrow$ $u = \int v\, dy \Rightarrow u = Cye^{-3x} + D$

Check:

$$\left.\begin{array}{l} 3u_y = Ce^{-3x} \\ u_{xy} = -3Ce^{-3x} \end{array}\right\} \Sigma = 0 \checkmark \qquad 3u_y =$$

6 Let u and v be any two solutions of the ODE.

Then $(u+v)''' - 3(u+v)'' + 4(u+v) = \underbrace{u''' - 3u'' + 4u}_{=0} + \underbrace{v''' - 3v'' + 4v}_{=0} = 0.$

and $\forall k \in \mathbb{R}$

$(ku)''' - 3(ku)'' + 4(ku) = k(\underbrace{u''' - 3u'' + 4u}_{=0}) = k \cdot 0 = 0$

So $u+v$ and ku are solutions, and the set of solutions is closed under addition & scalar multiplication.

The characteristic polynomial of the ODE is

$r^3 - 3r^2 + 4 = 0$, with roots $r = -1$, $r = 2$

Hence the solution set is spanned by $\{e^{-t}, e^{2t}, te^{2t}\}$ which has a non-zero Wronskian $(= 5e^{3t})$.

Since this set is linearly independent, it is a basis for the set of solutions (a f.s. of solutions).